THERMOPHYSICAL MEASUREMENTS UNDER MONOTONIC CONDITIONS

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UDC 536.2.08

Increasingly often, in practice, thermophysical measurements are made under quasisteady or monotonic conditions. These methods appear attractive to researchers because of their relative simplicity and because they allow combined measurements of the thermophysical properties (λ, a, and c) to be made over wide temperature ranges during continuous heating–cooling of a test specimen.

The theory of quasisteady-state methods, which originate from earlier works by Lykov, Ivantsov, and Adams [1, 2, 52], has been developed quite completely by now, in a number of monographs and review articles [3-10,* 11-15, 52-57]. At the same time, methods using monotonic conditions are relatively new and have not yet been described systematically in the technical literature.

Quasisteady-state methods, or methods of regular conditions of the second kind, are based on laws of the change of the temperature fields in bodies (test specimens), for constant rate of change of the ambient temperature ($t_a = t_0 + b\tau$) or under a constant thermal flux density at the surface $q(\tau)|_{x=R}$ = const. When these conditions are satisfied, the linear theory of heat conduction (λ, a, c = const) predicts that, after some instant of time, the temperature of every point in the body changes at a constant rate and the temperature field can be described by the equation:

$$\vartheta(r, \tau) = t(r, \tau) - t(0, \tau) = \frac{br^2}{2\Phi a} = \frac{qR}{2\lambda} \cdot \frac{r^2}{R^2}. \tag{1}$$

It follows from (1) that, in order to determine $a(t)$, it is necessary to measure the heating rate at any point and the temperature drop $\vartheta(r, \tau)$, while the determination of $\lambda(t)$ requires a measurement of thermal flux q and $\vartheta(r, \tau)$.

The practical implementation of quasisteady-state methods demands that the condition

$$b(r, \tau) = \text{const} \tag{2}$$

be ensured.

In measurements over a wide temperature range the thermophysical parameters generally do not remain constant, as required, and condition (2) can be ensured at only one point of the specimen. Automatic regulators used for this purpose add to the complexity of the measurement apparatus and reduce the reliability of experiments, especially in the high-temperature range.

Monotonic methods have evolved as a theoretical generalization and a practical extension of the quasisteady-state methods.

The basis of the method is an analysis of temperature fields in bodies during monotonic changes of temperature at variable rates. A most simple test mode is usually chosen: heating the specimen in an oven already brought up to temperature or heating it in an oven while the oven is brought up to temperature,

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*E. P. Shurygina, Methods of Determining the Thermal Constants of Hygroscopic Heat Insulators [in Russian], Candidate's Dissertation, Moscow Power Institute (1941).

Institute of Precision Mechanics and Optics, Leningrad. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 21, No. 4, pp. 750-760, October, 1971. Original article submitted August 8, 1970.

free cooling in a constant-temperature medium, monotonic heating with an internal heat source, etc. In some of these methods the computation formulas for $a(t)$, $\lambda(t)$, and $c(t)$ are derived by solving the nonlinear equation of heat conduction, taking account of the temperature dependence of $a(t)$, $\lambda(t)$, and heating rate $b(r, \tau)$ [26-46, 59].

The mathematical procedures for temperature-field analysis in monotonic measurements are diverse. The problem is solved by operational methods [32, 48], by expansion into power series [27, 28, 35, 38], by the "small parameter" method with successive approximation [37, 39-42], by integral substitutions [12, 36], by various approximate procedures [29, 30, 32, 49], and by other means [50, 51, 58].

Taits and Gol'dfarb take credit for the first attempt to prove the validity of monotonic thermophysical measurements [26]. Their proposed method of measuring the thermal diffusivity is based on the solution of the linear variant of the equation of heat conduction for a certain initial temperature drop ϑ_0 and specimen temperature rising at a constant rate. They represent the solution in the following form:

$$\frac{\vartheta}{b\tau} = \frac{1}{2\Phi \mathrm{Fo}}\left(1 - \frac{r^2}{R^2}\right) - \left(\frac{1}{\mathrm{Fo}} - \frac{2\Phi\vartheta_0}{b\tau}\right)\psi\left(\mathrm{Fo}, \frac{r}{R}\right). \tag{3}$$

In processing the test data, the $t = f(x, \tau)$ curve is subdivided into 80-100°C ranges within each of which the temperature appears to change linearly. For the calculation of a special graphs of $\vartheta/b\tau = f(\mathrm{Fo}, \vartheta_0/b\tau)$ are given; the Fo number and then the diffusivity a are determined from the graphs on the basis of test values for ϑ_0, b, and τ. The Taits–Gol'dfarb method has been used in practice for studying the thermophysical properties of large castings.

Kraev has developed methods of determining the thermal conductivity of heat insulators and metals, also of liquids, over the total temperature range from 20 to 700°C [27]. The interesting feature of these methods is, as the author indicates, that the heating (cooling) of a specimen may proceed under the constraint of any initial and any boundary conditions, provided only that the temperature of the specimen change monotonically with time. Theoretically, the Kraev methods are based on analyzing the nonlinear equation of heat conduction

$$\frac{1}{a}\cdot\frac{\partial t}{\partial \tau} = \left(\frac{\partial^2 t}{\partial r^2} + \frac{\Phi - 1}{r}\cdot\frac{\partial t}{\partial r}\right) + \frac{1}{\lambda}\cdot\frac{\partial \lambda}{\partial t}\left(\frac{\partial t}{\partial r}\right)^2, \tag{4}$$

the solution of which is sought in the form of a power series with coefficients varying as functions of time:

$$t\,(r, \tau) = A_0(\tau) + \sum_{n=1}^{\infty} A_{2n}(\tau)\, r^{2n}. \tag{5}$$

The coefficients in this series are determined by inserting (5) into Eq. (4) and its even-order derivatives at r = 0 [27]:

$$\begin{gathered} A_0\,(\tau) = t\,(0,\tau), \quad A_2 = \frac{1}{4a}\cdot\frac{dA_0}{d\tau} = \frac{1}{4a}\cdot\frac{dt_0}{d\tau}, \\ A_4 = \frac{1}{16}\cdot\frac{1}{a}\cdot\frac{dA_2}{d\tau} + \frac{1}{4}\cdot\frac{1}{c}\cdot\frac{dc}{dt}A_0^2 - \frac{1}{2\lambda}\cdot\frac{d\lambda}{dt}A_2^{2*} \text{ etc.} \end{gathered} \tag{6}$$

From (5) and (6) we can obtain a solution for $\vartheta(r, \tau) = t(r, \tau) - t(0, \tau)$ and a formula for calculating $a(t)$. In the case of a cylindrical specimen, the latter becomes

$$a(t_0) = \frac{R^2}{4\vartheta}\cdot\frac{dt_0}{d\tau}\left[1 + \frac{1}{4b_0}\cdot\frac{d\vartheta}{d\tau} + \frac{1}{4c}\cdot\frac{dc}{dt}\vartheta - \frac{1}{2\lambda}\cdot\frac{d\lambda}{dt}\vartheta\right]. \tag{7}$$

Expression (7) accounts for the temperature-dependence of the thermophysical properties and for the change in the temperature drop $\vartheta(r, \tau)$ with time. Kraev was the first to introduce a limit on the allowable magnitude of ϑ in a specimen. The author has shown that, if the test is performed at $\vartheta < 20$-30°C, the last terms inside the brackets of expression (7) may be disregarded and a few minutes after the test has begun the first correction term becomes quite small so that it too may be disregarded.

When the specimen is a plate, the formula for $a(t_0)$ derived by Kraev includes measurements of the time lag $\Delta\tau$ [27]:

$$a\,(t_0) = \frac{R^2}{2\Delta\tau}\left[1 - \frac{1}{3b_0}\cdot\frac{d^2 t_0}{d\tau^2}\Delta\tau - \frac{2}{3}\cdot\frac{1}{c}\cdot\frac{dc}{dt}\vartheta + \frac{1}{3}\cdot\frac{1}{\lambda}\cdot\frac{d\lambda}{dt}\vartheta\right]. \tag{8}$$

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* The derivation of (7) in [27] allows for an algebraic error in the determination of coefficient A_4. Hence we show the corrected version.

Pereletov has used an analogous analysis of the heat-conduction equation to establish the validity of a combined determination of a(t) and c(t) in the case of heat insulators [28]. He considers the temperature field of a monotonically heated hollow cylinder filled with the test material. The hollow cylinder acts as a calorimeter housing and is made of a material the properties of which are known. Only the nonlinearity of the temperature rise is taken into account in the solution of the problem, while the thermophysical properties of specimen and housing are assumed constant. Even in such a simplified version, however, the application of this analysis leads to difficult calculations and unwieldy formulas.

Brovkin has shown experimentally that the thermal diffusivity can be determined by heating a specimen with a monotonically decreasing thermal flux $q(\tau)$ [30]. No theoretical justification has been given for this, however.

The case of heating with a thermal flux monotonically changing with time at a constant rate, $q = q_0 \pm b\tau$, has been considered in [32, 58]. In [32] the problem was solved by the operational method. A formula was obtained for a(t) taking into account the relation $q = f(\tau)$:

$$a = \frac{R^2}{4\Delta\tau} \cdot \frac{1 + \sqrt{1 + b\Delta\tau/q}}{2}, \tag{9}$$

where q_{av} is the average value of q during the interval of time $\Delta\tau$. The method of estimating the correction proposed in [32] is not very convenient, since it requires knowledge of the time instant τ and the use of graphical differentiation. Cape, Lehman, and Nakata [59] have sought the solution to the analogous problem within a narrow time interval $\Delta\tau$ in the form of a power series:

$$t = \sum_{n=0}^{N} A_n V_n(r, \tau), \tag{10}$$

where A_n are constants. The functions $V_n(r, \tau)$ in the form of homogeneous polynomials of the n-th order with respect to r^2 and τ and the constants A_n are found by inserting (10) into the boundary conditions and into the equation of heat conduction. The formula in [54] for calculating a(t) is quite unwieldy in its structure and is not convenient for practical use.

In 1960, Platunov proposed a set of procedures for measuring a(t), c(t), and λ(t) for heat insulators and semiconductors [33]. They are based on the premise that the temperature of test specimens changes exponentially within narrow ranges of the $t(x, \tau)$ curve. This assumption becomes valid when specimens are heated in calorimetric devices with constant-power energy sources. Such a calorimeter consists of a massive metallic core with an effective heat-insulating shell. According to the theory of a regular heating mode, the temperature of such a system changes exponentially – to the first approximation – and the temperature field in the specimen during the regular heating period is described by the expression

$$t(x, \tau) - t(x, \infty) = -Au(x)\exp(-m\tau), \tag{11}$$

where $t(x, \tau)$ is the temperature of layer x at the instant of time τ, m is the heating rate, and u(x) is a function of the space coordinate. Formulas for calculating λ(t), c(t), and a(t) have been derived from (11) in [33]. Specifically, for calculating the thermal diffusivity during bilateral heating of a plate of thickness 2R we have the expression

$$a = \frac{R^2 b_0}{\Delta t_{-R,0} + \Delta t_{R,0}} f\left(\sqrt{\frac{m}{a}}R\right),$$

which for $mR^2/a < 0.01$ becomes identical to the known quasisteady-state relation.

In 1961-1962 Barskii extended his earlier method to the case where the temperature of a specimen changes monotonically [29]. He analyzed the errors of thermophysical measurements due to nonlinear changes in the surface temperature of a diathermal shell. The gist of the procedure is as follows. If at time $\tau = 0$, after a quasisteady state has been established, the heating rate $b_1 = dt_1/d\tau$ changes by db, then the temperature drop in the body will change proportionally to that change in the heating rate:

$$d\vartheta = A\frac{r^2}{R^2}db, \tag{12}$$

where A is some constant.

Introducing the concept of the system stabilization time τ^* as the time required for a body to reach a quasisteady state, Barskii finds the relative change of the temperature drop in a body during the time τ^*:

$$\delta(\vartheta)\,|_{\tau*} = \frac{\tau^*}{6b}\cdot\frac{db}{d\tau} \tag{13}$$

and the relative errors in the determination of λ, a, and c due to nonlinear temperature changes:

$$\delta(j) = z_j\tau^*\frac{db}{dE}, \quad j = \lambda,\ a,\ c. \tag{14}$$

In estimating the magnitude of τ^*, the author assumes that a quasisteady state has been reached when $Fo^* = 0.5$ and, for a plate, at $\tau^* = R^2/2a$.

This method of analysis is not sufficiently rigorous. According to Lykov's analysis of the problem in his monograph [3], the time to reach a quasisteady state depends on the kind of boundary conditions and by no means always corresponds to $Fo^* = 0.5$ (for example, $Fo^* = 0.4$ for a plate and q = const, $Fo^* = 1.8$ when $Bi \rightarrow \infty$ and $t_R = t_0 + b\tau$).

In 1962-1964 Platunov analyzed the temperature fields in a cylinder and in a sphere during monotonic heating with a combined consideration of the variable heating rate and of the temperature dependence of the thermophysical properties [37]. The fundamental assumption in this analysis was that the nonlinearities arising from these two considerations could be treated as perturbations.

The nonlinear equation of heat conduction (4) has been solved in [37] by the method of successive approximation. The thermophysical measurements were assumed here to have been performed at a small $\vartheta(r, \tau)$, usually not exceeding 30-100°C. Within such a range, provided that no phase transformations occur, the relations $\lambda(t)$, $a(t)$, $c(t)$, and $b(r, \tau)$ are approximated linearly as follows†

$$j = j_0(1 + k_j\vartheta), \text{where } j = a,\ \lambda,\ c,\ b. \tag{15}$$

Parameters j_0 and k_j are referred here to the base point temperature $t_0(\tau)$. The relative temperature coefficients k_j in (15) are functions of the temperature, but within the $\vartheta(r, \tau)$ ranges they are determined by the values of the respective parameters at $t_0(\tau)$.

The coefficients k_a, k_λ, and k_c are for most materials less than $(1\text{-}3)\cdot 10^{-3}$/°C and, therefore, the stipulation

$$|k_j\vartheta| < 0{,}1. \tag{16}$$

can rather easily be satisfied by a proper choice of ϑ.

Relations (15) are valid, moreover, within an error not greater than 1%.

Considering (15) and inequality (16), the nonlinear equation of heat conduction has been transformed in [37] to:

$$\frac{d^2\vartheta}{dr^2} + \frac{\Phi - 1}{r}\cdot\frac{d\vartheta}{dr} = \frac{b_0}{a_0}\left[1 - (k_a - k_b)\vartheta - k_\lambda\frac{a_0}{b_0}\left(\frac{d\vartheta}{dr}\right)^2\right], \tag{17}$$

where the terms inside the brackets are corrective, if the j = f(t) relations are recognized as having the effect of perturbations and as corresponding to (16). Equation (17) is then linearized on the basis of its zeroth-approximation solution (without the corrections, i. e., according to the laws of quasisteady heating). The solution to the linearized equation is found by conventional means. The temperature $t_0(\tau)$ and the gradient $d\vartheta/dr$ at the base point serve as the boundary constraints.

The expression

$$\vartheta(r, \tau) = \frac{b_0r^2}{2\Phi a}\left[1 + \frac{\Phi}{2(2+\Phi)}\left(k_b - k_a - \frac{2}{\Phi}k_\lambda\right)\frac{b_0r^2}{2\Phi a_0}\right]. \tag{18}$$

has been obtained in [37] for the three most simple body configurations.

The basic ideas of Platunov were confirmed in 1968 by Bezrukova, Sergeev, and Partskhaladze, who evaluated the effects of the temperature dependence of $\lambda(t)$, $a(t)$, $c(t)$, and $b(t)$ by the method of a small parameter [47].

When applied to thermophysical measurements, the method of successive approximations appears very effective for solving nonlinear problems. Subsequently, this method has been used for a thorough

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† Only a linear approximation is considered here. The method of successive approximation yields a solution for the general case, where $j(t) = j_0(1 + k_j\vartheta + n_j\vartheta + \ldots)$ [41].

analysis of temperature fields in flat, cylindrical, and spherical packing of a parallelepiped, a cube, or a cylinder with finite length during monotonic heating–cooling, and also for generalizing the laws which govern the regular heating mode of the first kind [39-42]. The results of this analysis have been used as a basis of a set of methods and devices for thermophysical studies over a wide total temperature range (from −150 to 3000°C) [43-46]. Lack of space permits us to dwell here only on a few general principles.

Expressions (18) applied to simple specimens become the basic ones from which computation formulas are derived. They agree with the results of [27, 47] obtained by other mathematical approaches. They yield the relations found in [31, 32, 48, 58] as special cases.

Depending on test conditions, these expressions may be put in a different form more convenient for practical use. Such a transformation can be effected with the aid of the relations between the relative temperature coefficients k_λ, k_a, k_c, k_b, $k_{b,\tau}$, and k_q in [41, 42, 48]:

$$k_\lambda = k_a + k_c, \quad k_b = k_{b,\tau} - k_a, \quad k_q = k_\lambda - k_a + k_{b,\tau}. \tag{19}$$

The first of these relations has been derived from the known relation $\lambda = ac\gamma$, the second relation has been derived from the equation of heat conduction in the zeroth approximation, and the third relation has been derived from the law of conservation by differentiating with respect to t and τ. Parameter $k_{b,\tau} = 1/b_0(db_0/dt)_\tau$, unlike k_b, characterizes the degree to which the function $t_0(\tau)$ deviates from linearity, and k_q characterizes the relative changes in the thermal flux density at the specimen surface. Obviously, it is easier to determine parameters $k_{b,\tau}$ and k_q experimentally than k_b.

The formulas for calculating the thermophysical properties by the monotonic methods can be derived from (18) directly, if the heating rate at the base point $b_0(\tau)$ and the temperature drop $\vartheta(r, \tau)$ are first measured:

$$a_0(t_0) = \frac{b_0 r^2}{2\Phi\vartheta}\left[1 + \frac{\Phi}{2(\Phi+2)}\left(k_{b,\tau} - 2k_a - \frac{2}{\Phi}k_\lambda\right)\vartheta\right]. \tag{20}$$

In order to determine the specific heat c(t) and the thermal conductivity λ(t), it is necessary in these tests to also measure the thermal flux density $q(\tau)$ absorbed by the specimen:

$$c(t_0) = \frac{\Phi q}{b_0\gamma_0 R}\left[1 - \frac{\Phi}{\Phi+2}(k_q - k_a)\vartheta(R, \tau)\right], \tag{21}$$

$$\lambda(t_0) = \frac{qR}{2\vartheta}\left[1 - \frac{\Phi}{2(\Phi+2)}\left(k_q - k_a + \frac{2+\Phi}{\Phi}k_\lambda\right)\vartheta\right]. \tag{22}$$

If any other point is chosen as the base, a surface point or a point at the bulk-mean temperature, then in Eqs. (20)-(22) only the structure of the correction terms will be different. The form of the correction can be found with the aid of relations (15) and simplified relations for the surface temperature drop ϑ_R or the bulk-mean temperature drop ϑ_V:

$$\vartheta_R = \vartheta(R, \tau) \simeq \frac{b_0 R^2}{2\Phi a_0}, \qquad \vartheta_V \simeq \frac{\Phi}{\Phi+2}\vartheta_R. \tag{23}$$

From the metrological standpoint, direct experimental measurement of $\vartheta(r, \tau)$ and $b_0(\tau)$ is not always convenient, while a sufficiently precise determination of the heating rate is technically difficult. For this reason, the quasisteady-state and the monotonic methods employ, in addition to the usual procedure, also a simpler procedure by which the time lag of the base-layer temperature τ_R behind the surface temperature is measured in the test [27, 32, 34]. The peculiarities of a quantitative transition from $\vartheta(r)$ and $b_0(\tau)$ values to τ_R values have been examined in [27, 39, 41, 42]. For this, the $t(r, \tau)$ function was expanded there into a Taylor series in τ_R around $t_0(\tau)$.

A transition to τ_R leads to a change in the structure as well as in the magnitude of the correction terms. Specifically, the formulas for calculating simple bodies become

$$a = \frac{R^2}{2\Phi\tau_R}\left[1 - \frac{1}{2+\Phi}(k_{b,\tau} - 2k_a + k_\lambda)\, b_0\tau_R\right]. \tag{24}$$

The structure of the correction terms for τ_R measurements in the case of more complicated monotonic heating modes is shown [41, 42].

The applicability limits for the computation formulas used in the monotonic methods are determined by inequalities (16) in combination with analogous stipulations for $k_q\vartheta$ and $k_{b,\tau}\vartheta$ [48]. Parameters k_λ, k_a, and k_c in these inequalities characterize the test material, while k_q and $k_{b,\tau}$ characterize the process and can, in principle, be varied by the experimenter. Stipulations (16) for k_λ, k_a, and k_c are usually satisfied for most materials at $\vartheta < 30\text{-}100°C$, while inequalities $|k_q\vartheta| < 0.1$ and $|k_{b,\tau}\vartheta| < 0.1$ are also valid for $|k_q| = |k_{b,\tau}| < (1\text{-}3) \cdot 10^{-3}/°C$. This means that in a monotonic mode one allows for a doubling of the rate $b(\tau)$ or of the thermal flux $q(\tau)$ per every 300-400°C of temperature change in the specimen. Such test conditions can be realized without automatic devices for maintaining $b_0(\tau) = \text{const}$ or $q = \text{const}$. Furthermore, such devices cannot, in principle, serve as universal means of ensuring the regularity of quasisteady-state conditions, since stipulations (16) for k_λ and k_a are governed by the allowable temperature drop in a specimen [48].

It is evident from an examination of the corrective terms in formulas (18), (20), (21), (22), and (24) that when

$$|k_j\vartheta| < \delta_{\text{add}} \tag{25}$$

with an error not greater than δ_{add} during monotonic heating–cooling, all laws governing a regular heating mode of the second kind are valid. Specifically, when $\vartheta < 5\text{-}10°C$ and $|k_j| < (1\text{-}3) \cdot 10^{-3}/°C$, the error $\delta_{\text{add}} < 1\%$. In this case, too, a doubling of the thermal flux $q(\tau)$ and of the rate $b_0(\tau)$ per 300-400°C during a test is allowed [48].

On the other hand, near phase transitions where the thermophysical properties change drastically, the coefficients k_λ, k_a, and k_c may have much higher values ($|k_j| > (3\text{-}5) \cdot 10^{-3}\,\text{deg}^{-1}$). Then, according to (16) and (25), it is necessary to include in the computation formulas the corrective terms for nonlinearity, and sometimes even higher-order approximations are required [41, 42].

One particular monotonic case is free heating or cooling of a body in a thermostatic medium ($t_a = \text{const}$). Such a test condition has been considered by Taits and Gol'dfarb in [31], neglecting the temperature dependence of λ, a, and α. With the aid of the laws governing a regular process of the first kind, they have shown that the time by which the temperature of an inner point lags behind the temperature of points on the periphery depends on the rate of heat transfer, i. e., on the Bi number. For a point at the bulk-mean temperature this dependence is weaker than for a point on the surface:

$$\tau_{\text{msurf}} = \frac{R^2}{2\Phi a}\,\varphi(\text{Bi}) \text{ and } \tau_{\text{mvol}} = \frac{R_v^2}{2\Phi a}\,\psi(\text{Bi}), \qquad \varphi(\text{Bi}) > \psi(\text{Bi}). \tag{26}$$

When $\text{Bi} < 0.2$, then $\varphi(\text{Bi}) = \psi(\text{Bi}) = 1$ with an error not exceeding 1%.

A more general case of free cooling has been considered in [40]. It is shown there that, if conditions

$$|k_\alpha\vartheta| < 0{,}1 \text{ and } |\vartheta/\theta| < 0.1 \tag{27}$$

are satisfied, where θ denotes the temperature excess of the body surface above the surrounding medium, all the laws governing a monotonic process apply to free cooling. Using the equalities $q = \alpha\theta$ and $q \simeq 2(\lambda\vartheta/R)$, the second inequality in (27) may be expressed differently:

$$\text{Bi} < 0.2, \tag{28}$$

and this is in complete agreement with the conclusions arrived at in [31].

The initial period of a quasisteady or a monotonic process is without significance and, as a rule, not taken into consideration. The experimenter must know the length of its duration, however, in order properly to disregard it in the analysis. In the quasisteady-state methods the duration of the transient can be determined by solving the appropriate Lykov problems, i. e., by evaluating the relative effect of discarded terms on the regular component of $\vartheta(r, \tau)$ [3]. The magnitude of the relative error δ_{add} incurred by disregarding these terms depends on the form of the boundary conditions. Specifically, $\text{Fo}^* = 0.4$ with an error $\delta_{\text{add}} = 1\%$ for a plate with $q = \text{const}$ and $\text{Fo}^* = 1.9$ for $t_R = t_0 + b\tau$ [3].

The length of this insignificant period in a monotonic process is determined by stipulation (25) for k_b. However, its practical evaluation in an actual test is difficult. This problem has not been solved rigorously and in sufficient simple terms for $\lambda(t)$, $a(t)$, and $b(r, \tau)$, even with an assumed uniform initial distribution of these parameters.

Pak and Osipova in [36] have based their method of measuring the thermal diffusivity on the temperature dependence of $\lambda(t)$ and $c(t)$ and on an initially uniform distribution. For linearizing the equation of heat conduction (4) they used the following integral substitutions:

$$v = \frac{1}{\lambda_0} \int_0^t \lambda(t)\,dt \quad \text{and} \quad \xi = \frac{1}{a_0} \int_0^\tau a(t)\,dt. \tag{29}$$

Under specific conditions this substitution yields the equation

$$\frac{\partial v}{\partial \xi} = a_0 \left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial v}{\partial r} \right), \tag{30}$$

which has an exact solution.

The formulas for calculating $a(t)$ and $\lambda(t)$ for a cylindrical specimen are obtained in the following form:

$$\begin{gathered} a = \frac{r^2}{4\Delta\tau} \left(1 + \int_0^\tau N\vartheta(\tau)\,d\tau \right), \\ \lambda = \frac{qr^2}{2R\vartheta} \left(1 + \frac{N\Delta\tau}{\vartheta} \right). \end{gathered} \tag{31}$$

The authors show that "at small temperature drops within the specimen and at small heating rates the correction terms in Eq. (31) may be omitted."

In [48] a solution to the problem of heating a plate or a cylinder with a monotonically changing thermal flux $q(\tau)$ has been obtained for the case in which λ = const and a = const. It was assumed there that the $q(\tau)$ relation could be represented as a polynomial:

$$q(\tau) = \sum_{i=0}^{n} q_i \tau^i. \tag{32}$$

The expressions for $\vartheta(r, \tau)$ during the regular-process period, which have been derived in [48] with the aid of (19) and (23), agree with the results in [27, 37, 44]. The following expressions have been obtained in [48] for evaluating the length of the initial transient period when $|k_q\vartheta| < 0.2\text{-}0.3$ for a cylinder or for a plate respectively:

$$\tau_{\text{pre}} = \frac{R_1^2/a}{\mu_1^2 + 4k_q\vartheta_R} \ln \frac{0.95}{\delta_{\text{add}}}, \quad \mu_1^2 = 14.4, \tag{33}$$

$$\tau_{\text{pre}} = \frac{R^2/a}{\eta_1^2 + 2k_q\vartheta_R} \ln \frac{0.8}{\delta_{\text{add}}}, \quad \eta_1^2 = \pi^2. \tag{34}$$

Thus, the theory on which the monotonic heating–cooling methods are based has been well developed by now.

These methods facilitate large-scale thermophysical measurements on metals, heat insulators, heat semiconductors, and ceramics within the −150 to 3000°C temperature range. All the prerequisites exist for the wide applicability of these methods in the study of thermophysical properties of gases and liquids, especially in the high temperature and pressure ranges. The measurement error incurred by these methods is, as a rule, determined by their degree of sophistication as well as by the design of the calorimetric instruments, and it usually does not exceed the error of classical methods.

NOTATION

a	is the thermal diffusivity;
λ	is the thermal conductivity;
c	is the true specific heat;
$t(r, \tau)$	is the temperature of a body at a point of coordinate r at the instant of time τ;
$\vartheta(r, \tau) = t(r, \tau) - t(0, \tau)$	is the temperature drop relative to the point of coordinate $r = 0$;
$b = b(r, \tau) = dt/d\tau$	is the rate of temperature change at the point r;
$q = q(\tau)$	is the thermal flux density at the body surface;
2R	is the thickness of a plate, the diameter of a cylinder;
Φ	is a constant equal to 1, 2, 3 for a plate, a cylinder, and a sphere, respectively;
$\text{Fo} = a\tau/R^2$	is the Fourier number;
$\text{Bi} = \alpha R/\lambda$	is the Biot number;

α	is the heat-transfer coefficient;
$\Delta\tau$, τ_R	is the time by which the temperature of a base point lags behind that of peripheral points;
$k_j = (1/j_0)(dj/dt)$, $j(t) = a, c, \lambda, \alpha, b, q$	are the relative temperature coefficients of the respective thermophysical parameters;
m	is the rate of heating the system;
δ	is the relative error;
γ	is the density;
$t_0 = t(0, \tau)$;	
$b_0 = b(0, \tau)$;	
t_a	is the ambient temperature;
$\theta = t(r, \tau) - t_a$	is the temperature excess of the body surface above the surrounding medium;
τ_{pre}	is the length of the initial transient period before regular process conditions are reached in a test.

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